The Origin of Geometry

by Michel Serres

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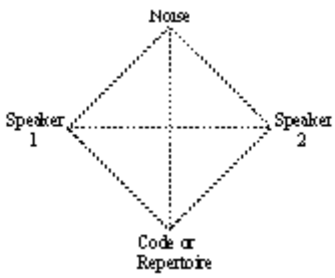
Renan had the best reasons in the world for calling the advent of mathematics in Greece a miracle. The construction of geometric idealities or the establishment of the first proofs were, after all, very improbable events. If we could form some idea of what took place around Thales and Pythagoras, we would advance a bit in philosophy. The beginnings of modern science in the Renaissance are much less difficult to understand; this was, all things considered, only a reprise. Bearing witness to this Greek miracle, we have at our disposal two groups of texts. First, the mathematical corpus itself, as it exists in the *Elements* of Euclid, or elsewhere, treatises made up of fragments. On the other hand, doxography, the scattered histories in the manner of Diogenes Laertius, Plutarch, or Athenaeus, several remarks of Aristotle, or the notes of commentators such as Proclus or Simplicius. It is an understatement to say that we are dealing here with two groups of texts; we are in fact dealing with *two languages*. Now, to ask the question of the Greek beginning of geometry is precisely to ask how one passed from one language to another, from one type of writing to another, from the language reputed to be natural and its alphabetic notation to the rigorous and systematic language of numbers, measures, axioms, and formal arguments. What we have left of all this history presents nothing but two languages as such, narratives or legends and proofs or figures, words and formulas. Thus it is as if we were confronted by two parallel lines which, as is well known, never meet. The origin constantly recedes, inaccessible, irretrievable. The problem is open.

I have tried to resolve this question three times. First, by immersing it in the technology of communications. When two speakers have a dialogue or a dispute, the channel that connects them must be drawn by a diagram with four poles, a complete square equipped with its two diagonals. However loud or irreconcilable their quarrel, however calm or tranquil their agreement, they are linked, in fact, twice: they need, first of all, a certain intersection of their repertoires, without which they would remain strangers; they then band together against the noise which blocks the communication channel. These two conditions are necessary to the diaIogue, though not sufficient. Consequently, the two speakers have a common interest in excluding a third man and including a fourth, both of whom are prosopopoeias of the,powers of noise or of the instance of intersection. (1)Now this schema functions in exactly this manner in Plato's *Dialogues*, as can easily be shown, through the play of people and their naming, *their resemblances and differences*, their mimetic preoccupations and the dynamics of their violence. Now then, and above all, the mathematical sites, from the *Meno* through the *Timaeus*, by way of the *Statesman* and others, are all reducible geometrically to this diagram. Whence the origin appears, we pass from one language to another, the language said to be natural presupposes a dialectical schema, and this schema, drawn or written in the sand, as such, is the first of the geometric idealities. Mathematics presents itself as a successful dialogue or a communication which rigorously dominates its repertoire and is maximally purged of noise. Of course, it is not that simple. The irrational and the unspeakable lie in the details; listening always requires collating; there is always a leftover or a residue, indefinitely. But then, the schema remains open, and history possible. The philosophy of Plato, in its presentation and its models, is therefore inaugural, or better yet, it seizes the inaugural moment.

To be retained from this first attempt at an explanation are the expulsions and the purge. Why the parricide of old father Parmenides, who had to formulate, for the first time, the principle of contradiction. To be noted here again is how two speakers, irreconcilable adversaries, find themselves forced to turn together against the same third man for the dialogue to remain possible, for the elementary link of human relationships to be possible, for geometry to become possible. Be quiet, don't make any noise, put your head back in the sand, go away or die. Strange diagonal which was thought to be so pure, and which is agonal and which remains an agony.



The second attempt contemplates Thales at the foot of the Pyramids, in the light of the sun. It involves several geneses, one of which is ritual.(2) But I had not taken into account the fact that the Pyramids are also tombs, that beneath the theorem of Thales, a corpse was buried, hidden. The space in which the geometer intervenes is the space of similarities: he is there, evident, next to three tombs of the *same* form and of *another* dimension -the tombs are *imitating* one another. And it is the pure space of geometry, that of the group of *similarities* which appeared with Thales. The result is that the theorem and its immersion in Egyptian legend says, without saying it, that there lies beneath the *mimetic operator*, constructed concretely and represented theoretically, a hidden royal corpse. I had seen the sacred above, in the sun of Ra and in the Platonic epiphany, where the sun that had come in the ideality of stereometric volume finally assured its diaphaneity; I had not seen it below, hidden beneath the tombstone, in the incestuous cadaver. But let us stay in Egypt for a while.

The third attempt consists in noting the double writing of geometry.(3) Using figures, schemas, and diagrams. Using letters, words, and sentences of the system, organized by their own semantics and syntax. Leibniz had already observed this double system of writing, consecrated by Descartes and by the Pythagoreans, a double system which represents itself and expresses itself one by the other. He sometimes liked, as did many others, to privilege the intuition, clairvoyant or blind, required by the first [diagrams] over the deductions produced by the second [words]. There are, as is well known, or as usual, two schools of thought on the subject. It happens that they trade their power throughout the course of history. It also happens that the schema contains more information than several lines of writing, that these lines of writing lay out indefinitely what we draw from the schema, as from a well or a cornucopia. Ancient algebra writes, drawing out line by line what the figure of ancient geometry dictates to it, what that figure contains in one stroke. The process never stopped; we are still talking about the square or about the diagonal. We cannot even be certain that history is not precisely that.

Now, many histories report that the Greeks crossed the sea to educate themselves in Egypt. Democritus says it; it is said of Thales; Plato writes it in the *Timaeus*. There were even, as usual, two schools at odds over the question. One held the Greeks to be the teachers of geometry; the other, the Egyptian priests. This dispute caused them to lose sight of the essential: that the Egyptians wrote in ideograms and the Greeks used an alphabet. Communication between the two cultures can be thought of in terms of the relation between these two scriptive systems (*signaletiques*). Now, this relation is precisely the same as the one in geometry which separates and unites figures and diagrams on the one hand, algebraic writing on the other. Are the square, the triangle, the circle, and the other figures all that remains of hieroglyphics in Greece? As far as I know, they are ideograms. Whence the solution: the historical relation of Greece to Egypt is thinkable in terms of the relation of an alphabet to a set of ideograms, and since geometry could not exist without writing, mathematics being written rather than spoken, this relation is brought back into geometry as an operation using a double system of writing. There we have an easy passage between the natural language and the new language, a passage which can be carried out on the multiple condition that we take into consideration two different languages, two different writing systems and their common ties. And this resolves in tum the historical question: the brutal stoppage of geometry in Egypt, its freezing, its crystallization into fixed ideograms, and the irrepressible development, in Greece as well as in our culture, of the new language, that inexhaustible discourse of mathematics and rigor which is the very history of that culture. The inaugural relation of the geometric ideogram to the alphabet, words, and sentences opens onto a limitless path.

This third solution blots out a portion of the texts. The old Egyptian priest, in the *Timaeus*, compares the knowledge of the Greeks when they were children to the time-wom science of his own culture.(4) He evokes, in order to compare them, floods, fires, celestial fire, catastrophes. Absent from the solution are the priest, history, either mythical or real, in space and time, the violence of the elements which hides the origin and which, as the *Timaeus* clearly says, always hides that origin. Except, precisely, from the priest, who knows the secret of this violence. The sun of Ra is replaced by Phaethon, and mystical contemplation by the catastrophe of deviation.

We must start over -go back to those parallel lines that never meet. On the one hand, histories, legends, and doxographies, composed in natural language. On the other, a whole corpus, written in mathematical signs and symbols by geometers, by arithmeticians. We are therefore not concerned with merely linking two sets of texts; we must try to glue, two languages back together again. The question always arose in the space of the relation between experience and the abstract, the senses and purity. Try to figure out the status of the pure, which is impure when history changes. No. Can you imagine (that there exists) a Rosetta Stone with some legends written on one side, with a theorem written on the other side? Here no language is unknown or undecipherable, no side of the stone causes problems; what is in question is the edge common to the two sides, their common border; what is in question is the stone itself.

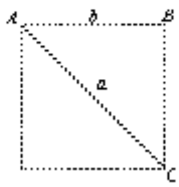
Legends. Somebody or other who conceived some new solution sacrificed an ox, a bull. The famous problem of the duplication of the cube arises regarding the stone of an altar at Delos. Thales, at the Pyramids, is on the threshold of the sacred. We are not yet, perhaps, at the origins. But, surely, what separates the Greeks from their possible predecessors, Egyptians or Babylonians, is the establishment of a proof. Now, the first proof we know of is the apagogic proof on the irrationality of $\sqrt{2}$. (5)

And so, legends, once again. Euclid's *Elements*, Book X, first scholium. It was a Pythagorean who proved, for the first time, the so-called irrationality [of numbers]. Perhaps his name was Hippasus of Metapontum. Perhaps the sect had sworn an oath to divulge nothing. Well, Hippasus of Metapontum spoke. Perhaps he was expelled. In any case, it seems certain that he died in a shipwreck. The anonymous scholiast continues: "The authors of this legend wanted to speak through allegory. Everything that is irrational and deprived of form must remain hidden, that is what they were trying to say. That if any soul wishes to penetrate this secret region and leave it open, then it will be engulfed in the sea of becoming, it will drown in its restless currents."

Legends and allegories and, now, history. For we read a significant event on three levels. We read it in the scholia, commentaries, narratives. We read it in philosophical texts. We read it in the theorems of geometry. The event is the *crisis*, the famous crisis of irrational numbers. Owing to this crisis, mathematics, at a point exceedingly close to its origin, came very close to dying. In the aftermath of this crisis, Platonism had to be recast. The crisis touched the logos. If logos means proportion, measured relation, the irrational or alogon is the impossibility of measuring. If logos means discourse, the alogon prohibits speaking. Thus exactitude crumbles, reason is mute.

Hippasus of Metapontum, or another, dies of this crisis, that is the legend and its allegorical cover in the scholium of the *Elements*. Parmenides, the father, dies of this crisis-this is the philosophical sacrifice perpetrated by Plato. But, once again, history: Plato portrays Theaetetus dying upon returning from the the battle of Corinth (369), Theaetetus, the founder, precisely, of the theory of irrational numbers as it is recapitulated in Book X of Euclid. The crisis read three times renders the reading of a triple death: the legendary death of Hippasus, the philosophical parricide of Parmenides, the historical death of Theaetetus. One crisis, three texts, one victim, three narratives. Now, on the other side of the stone, on the other face and in another language, we have the crisis and the possible death of mathematics in itself.

Given then a proof to explicate as one would a text. And, first of all, the proof, doubtless the oldest in history, the one which Aristotle will call *reduction to the absurd*. Given a square whose side $AB = b$, whose diagonal $AC = a$:



We wish to measure AC in terms of AB. If this is possible, it is because the two lengths are mutually commensurable. We can then write $AC/AB = a/b$. It is assumed that a/b is reduced to its simplest form, so that the integers a and b are mutually prime. Now, by the Pythagorean theorem: $a^2 = 2b^2$. Therefore a^2 is even, therefore a is even. And if a and b are mutually prime, *b is an odd number*. If a is even, we may posit: $a = 2c$. Consequently, $a^2 = 4c^2$. Consequently $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. Thus, *b is an even number*.

The situation is intolerable, the number *b is at the same time even and odd*, which, of course, is impossible. Therefore it is impossible to measure the diagonal in terms of the side. They are mutually incommensurable. I repeat, if logos is the proportional, here a/b or $1/\sqrt{2}$, the alogon is the incommensurable. If logos is discourse or speech, you can no longer say anything about the diagonal and $\sqrt{2}$ is irrational. It is impossible to decide whether b is even or odd. Let us draw up the list of the notions used here. 1) What does it mean for two lengths to be mutually commensurable? It means that they have common aliquot parts. There exists, or one could make, a ruler, divided into units, in relation to which these two lengths may, in turn, be divided into parts. In other words, they are *other* when they are alone together, face to face, but they are *same*, or just about, in relation to a third term, the unit of measurement taken as reference. The situation is interesting, and it is well known: *two irreducibly different entities are reduced to similarity through an exterior point of view*. It is fortunate (or necessary) here that the term *measure* has, traditionally, at least two meanings, the geometric or metrological one and the meaning of non-disproportion, of serenity, of nonviolence, of peace. These two meanings derive from a similar situation, an identical operation. Socrates objects to the violent crisis of Callicles with the famous remark: you are ignorant of geometry. The Royal Weaver of the *Statesman* is the bearer of a supreme science: superior metrology, of which we will have occasion to speak again. 2) What does it mean for two numbers to be mutually prime? It means that they are radically different, that they have no common factor besides one. We thereby ascertain the first situation, their total otherness, unless we take the unit of measurement into account. 3) What is the Pythagorean theorem? It is the fundamental theorem of measurement in the space of *similarities*. For it is invariant by variation of the coefficients of the squares, by variation of the forms constructed on the hypotenuse and the two sides of the triangle. And the space of similarities is that space where things can be of the *same* form and of *another* size. It is the space of models and of imitations. The theorem of Pythagoras founds measurement on the representative space of imitation. Pythagoras sacrifices an ox there, repeats once again the legendary text. 4) What, now, is evenness? And what is oddness? The English terms reduce to a word the long Greek discourses: *even* means equal, united, flat, *same*; *odd* means bizarre, unmatched, extra, left over, unequal, in short, *other*. To characterize a number by the absurdity that it is at the same time even and odd is to say that it is at the same time *same* and *other*.

Conceptually, the apagogic theorem or proof does *nothing but* play variations on the notion of same and other, using measurement and commensurability, using the fact of two numbers being-mutually prime, using the Pythagorean theorem, using evenness and oddness. It is a rigorous proof, and the first in history, based on *mimesis*. It says something very simple: *supposing mimesis, it is reducible to the absurd*. Thus the crisis of irrational numbers overturns Pythagorean arithmetic and early Platonism.

Hippasus revealed this, he dies of it -end of the first act.

It must be said today that this was said more than two millennia ago. Why go on playing a game that has been decided? For it is as plain as a thousand suns that if the diagonal or $\sqrt{2}$ are incommensurable or irrational, they can still be constructed on the square, that the mode of their geometric existence is not different from that of the side. Even the young slave of the *Meno*, who is ignorant, will know how, will be able, to construct it. In the same way, children know how to spin tops which the *Republic* analyzes as being stable and mobile at the same time. How is it then that reason can take facts that the most ignorant children know how to establish and construct, and can demonstate them to be irrational? There must be a reason for this irrationality itself.

In other words, we are demonstrating the absurdity of the irrational. We reduce it to the contradictory or to the undecidable. Yet, it exists; we cannot do anything about it. The top spins, even if we demonstrate that, for impregnable reasons, it is, undecidably, both mobile and fixed. That's the way it is. *Therefore*, all of the theory which precedes and founds the proof must be reviewed, transformed. It is not reason that governs, it is the obstacle. What becomes absurd is not what we have proven to be absurd, it is the theory on which the proof depends. Here we have the very ordinary movement of science: once it reaches a dead-end of this kind, it immediately transforms its presuppositions.

Translation: *mimesis* is reducible to contradiction or to the undecidable. Yet it exists; we cannot do anything about it. It spins. It works, as they say. That's the way it is. It can always be shown that we can neither speak nor walk, or that Achilles will never catch up with the tortoise. Yet, we do speak, we do walk, the fleet-footed Achilles does pass the tortoise. That's the way it is. *Therefore*, all of the theory which precedes must be transformed. What becomes absurd is not what we have proven to be absurd, it is the theory as a whole on which the proof depends.

Whence the (hi)story which follows. Theodorus continues along the legendary path of Hippasus. He multiplies the proofs of irrationality. He goes up to $\sqrt{17}$. There are a lot of these absurdities, there are as many of them as you want. We even know that there are many more of them than there are of rational relations. Whereupon Theaetetus takes up the archaic Pythagoreanism again and gives a general theory which grounds, in a new reason, the facts of irrationality. Book X of the *Elements* can now be written. The crisis ends, mathematics recovers an order, Theaetetus dies, here ends this story, a technical one in the language of the system, a historical one in the everyday language that relates the battle of Corinth. Plato recasts his philosophy, father Parmenides is sacrificed during the parricide on the altar of the principle of contradiction; for surely the *Same* must be *Other*, after a fashion. Thus, Royalty is founded. The Royal Weaver combines in an ordered web rational proportions and the irrationals; gone is the crisis of the reversal, gone is the technology of the dichotomy, founded on the square, on the iteration of the diagonal. Society, finally, is in order. This dialogue is fatally entitled, not *Geometry*, but the *Statesman*.

The Rosetta Stone is constructed. Suppose it is to be read on all of its sides. In the language of legend, in that of history, that of mathematics, that of philosophy. The message that it delivers passes from language to language. The crisis is at stake. This crisis is sacrificial. A series of deaths accompanies its translations into the languages considered. Following these sacrifices, order reappears: in mathematics, in philosophy, in history, in political society. The schema of Rene Girard allows us not only to show the isomorphism of these languages, but also, and especially, their link, how they fit together.(6) For it is not enough to narrate, the operators of this movement must be made to appear. Now these operators, all constructed on the pair Same-Other, are seen, deployed in their rigor, throughout the very first geometric proof. just as the square equipped with its diagonal appeared, in my first solution, as the thematized object of the complete intersubjective relation, formation of the ideality as such, so the rigorous proof appears as such, manipulating all the operators of *mimesis*, namely, the internal dynamics of the schema proposed by Girard. The origin of geometry is immersed in sacrifical history and the two parallel lines are henceforth in connection. Legend, myth, history, philosophy, and pure science have common borders over which a unitary schema builds bridges.

Metapontum and geometer, he was the Pontifex, the Royal Weaver. His violent death in the storm, the death of Theaetetus in the violence of combat, the death of father Parmenides, all these deaths are murders. The irrational is mimetic. The stone which we have read was the stone of the altar at Delos. And geometry begins in violence and in the sacred.(7)

Notes:

(1) The line from Speaker 1 to Speaker 2 represents the channel of communication thatjoins the two speakers together. The line from Noise to the Code or Repertoire represents the indissoluble link between noise and the code. Noise always threatens to overwhelm the code and to disrupt communication. Successful communication, then, requires the exclusion of a third term (noise) and the inclusion of a fourth (code). See "Platonic Dialogue," chapter 6 of' the present volume. See also Michel Serres. Le Parasite (Paris: Grasset, 1980). -Ed.

(2) See "Mathematics and Philosophy: What Thales Saw...... chapter 8 of the present volume. -Ed.

(3)This third explaiiatioii appears as "Origine de la geometrie, 4" in Michel Serres, Hermes V.- Le Passage du Nord-Ouest (Paris: Minuit, 1980), pp. 175-84. -Ed.

(4) Plato, Timaeus, 22b ff.

(5) An apagogic proof is one that proceeds by disproving the proposition which contradicts the one to be established, in other words, that proceeds by reductio ad absurduni. - Ed.

(6) The reference is to Rene Girard's theory of the emissary victim. See chapter 9, note 9 in the present volume. - Ed.

(7) It is just as remarkable that the physics of Epicurus, as Lucretius develops it in De Rerum Natura, is framed by the sacrifice of Iphigenia and the plague of Athens. These two events, legendary or historical, can be read using the grid of phvsics. But, inversely, all this physics can be read using the same schema, since the term inane means "purge" and "expulsion." I have shown this in detail in La Naissance de la physique dans le texte de Luctice: Fleuves el turbulences (Paris: Minuit, 1977). (See also "Lucretius: Science and Religion," chapter 9 of the present volume. -Ed.)